TAKING INDUSTRY STRUCTURING SERIOUSLY:
A STRATEGIC PERSPECTIVE ON PRODUCT DIFFERENTIATION†
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Given legal impediments to consolidation and collusion, firms often resort to product differentiation to attain market power. This paper provides a formal analysis of product differentiation as a tool for such industry structuring at both the firm and industry level. We examine: how industry structure differs when firms collaborate on their differentiation decisions, and when the profitability of such collaboration is greatest; how an individual firm’s differentiation decisions affect subsequent market outcomes under price competition, such as margin, market share, and profit; how mere differentiation differs from a ‘differentiation advantage’; and how changing a firm’s differentiation affects its rivals through both positive externalities (by restraining rivalry) and negative externalities (by shifting competitive advantage). Our results have implications for empirical research, strategy theory, and pedagogy. Copyright © 2012 John Wiley & Sons, Ltd.

INTRODUCTION
Product differentiation as a tool for industry structuring

For decades, the economics discipline has studied how an industry’s structure affects outcomes like the conduct and performance of its firms, the utility of its customers, and the public welfare of the economy (Bain, 1951, 1959), and the strategy field has benefitted from importing this knowledge as a tool for analyzing industries (Porter, 1979). Yet, we know far more about how industry structure affects firms than about how firms, either individually or jointly, affect their industry’s structure. Exactly how should firms ‘alter [the] causes’ of their industry’s structure (Porter, 1979: 144), and be ‘smart competitors’ (Yoffie, 1994: 12) who collaborate to shape that structure? Analysis

‘When dealing with the forces that drive industry competition, a company can devise a strategy that takes the offensive. This posture is designed to do more than merely cope with the forces themselves; it is meant to alter their causes... The balance of forces is partly a result of external factors and partly in the company’s control.’ – Porter (1979: 144)

‘Coke and Pepsi did not just inherit this business; they created it. Part of their ongoing success will be a function of their abilities to structure not only their own businesses, but the industry as a whole. In other words, industry structure is not always exogenous... it can be endogenous. Coke and Pepsi are... ‘smart’ competitors – when

they go to war, they kill the bystanders, not themselves.’ – Yoffie (1994: 12)

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of these issues from a *strategic*, rather than an *economic*, perspective is lacking. With the strategy field’s mandate to study how firms actively boost performance, this strategic perspective should focus more on industry structuring (verb) than on industry structure (noun).

Overt methods of industry structuring include consolidation by acquiring competitors, collusion on prices or market shares, and exclusive contracts that ‘assign each buyer to a single seller’ (Stigler 1964: 46). Each of these overt methods has occasionally been authorized by law: The 1913 Kingsbury Commitment legitimated telecommunications consolidation by AT&T, the 1938 Civil Aeronautics Act supported price collusion by airlines, and the 1980 Soft Drink Interbrand Competition Act ratified exclusive bottling contracts. However, such overt methods are usually prohibited by antitrust laws.

Without these overt methods, the goal of assigning each buyer to a single seller can be approximated via product differentiation, wherein each seller commits to sell a different version of the product than its rivals. If buyers’ preferences differ from each other, then differentiation exploits those differences by inducing each buyer to voluntarily *self-assign* him or herself to whichever seller’s version of the product most closely matches his or her preferences. Moreover, whereas colluding directly on prices and market shares ‘is usually an easy form of collusion to detect’ (Stigler, 1964: 46), cooperating on differentiation may be more difficult to detect by antitrust authorities, who may also be reluctant to interfere in matters of firm strategy. (Imagine regulators ordering Ryanair to improve the comfort of its air travel service to be less differentiated from British Airways, or vice versa.) In this paper, we ask: how would industry structure and market outcomes differ if ‘smart competitors’ cooperate in this way? Does the answer depend on the type of differentiation? When would such cooperative industry structuring be most profitable? Why?

Answering these questions requires understanding precisely how differentiation affects profit. In that connection, since Demsetz’s (1973, 1974) critique of antitrust economics, researchers have recognized that firms can take two distinct approaches toward competition to increase profit. One approach is to restrain their rivalry with each other to soften price competition. The other approach is to exploit a competitive advantage to create more economic value than rivals, where ‘economic value’ is the gap between customers’ willingness to pay for a product and the firm’s cost to provide the product (MacDonald and Ryall, 2004; Peteraf and Barney, 2003); this superior value creation allows the firm to dominate its market and thereby profit at the expense of competitors. These two approaches, labeled here as rivalry restraint and competitive advantage, respectively, underpin each other’s effectiveness, because rivalry restraint requires accommodative behavior toward rivals to prevent competition from escalating in intensity, whereas maximizing the value of a competitive advantage requires aggressive behavior toward rivals. For this reason, economists have recognized that rivalry is more difficult to restrain in industries where a firm has a competitive advantage (Bain, 1948; Jacquemin and Slade, 1989; Schmalensee, 1987), and researchers are investigating how this inconsistency affects strategy (Chatain and Zemsky, 2011; Makadok, 2010, 2011).

Product differentiation affects both rivalry restraint and competitive advantage. This dual nature is reflected in its appearance in two different parts of typical strategy textbooks: (1) chapters on industry analysis, where differentiation drives several of the ‘five forces,’ and (2) chapters on competitive advantage, where a ‘differentiation advantage’ is distinguished from low cost leadership. On the one hand, product differentiation in industry analysis (Porter, 1979) operates via rivalry restraint: the more that the products of rival firms differ, the more strongly any given consumer will prefer one firm’s product over another’s and so the less effective price cutting will be at gaining market share, which in turn restrains price rivalry and thereby raises the whole industry’s profits. On the other hand, a firm with a differentiation advantage creates more economic value than its rivals, enabling it to boost its own profit at their expense.

This paper uses formal modeling to analyze the inherently *strategic* implications of product differentiation, as distinct from its economic or marketing implications. This requires taking both industry-level and firm-level perspectives. At the industry level, we assume that firms are symmetric *ex ante* and then examine how they differentiate

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1Porter (1979) also notes that product differentiation may dampen customer bargaining power, preventing customers from playing rivals off against each other, and may foster customer loyalty, which also restrains rivalry from entrants or substitutes.
in anticipation of subsequent price competition. Unlike other models of differentiation, we compare how the level and nature of differentiation differ between two cases: where each firm makes its positioning choices independently to maximize its own profit, and where they cooperate on their positioning as ‘smart competitors’ who maximize their joint profit. Given the symmetric starting point, we are especially interested in the conditions under which asymmetries between firms emerge endogenously within the model. Our firm-level analysis takes firms’ prior differentiation decisions—whether symmetric or asymmetric—as given, and then considers how a unilateral change in differentiation by one firm affects price competition via both rivalry restraint and competitive advantage.

To this end, we adapt, extend, and reanalyze two canonical economic models, one of horizontal differentiation (D’Aspremont, Gabszewicz, and Thisse, 1979; Hotelling, 1929) and one of vertical differentiation (Mussa and Rosen, 1978), which differ as follows:

If we consider a class of goods as being typified by a set of (desirable) characteristics, then two varieties are vertically differentiated when the first contains more of some or all characteristics than the second, so that all rational consumers given a free choice would opt for the first. They are horizontally differentiated when one contains more of some but fewer of other characteristics, so that two consumers exhibiting different tastes offered a free choice would not unambiguously plump for the same one. (Waterson, 1989: 2).

So, vertical differentiation captures a product’s quality while horizontal differentiation captures its qualities. We study both to learn whether there are strategic, as opposed to economic differences between them.²

In our horizontal model, two firms compete in a market where customers differ in their ideal level of a particular product attribute. For instance, some consumers may prefer a sweeter soda, whereas others prefer one less sweet; or some consumers may prefer a restaurant on the west side of town, whereas others prefer one on the east side (or in the city center). As in all such models, horizontal differentiation has two effects: first, it moves the firm away from its rival, which restrains price rivalry; second, although it moves the firm closer to customers at one end of the market who may view the product as more attractive, it also moves the firm farther from a majority of the market, so most customers view its product as less attractive. The combined result is a net competitive disadvantage.

In our vertical model, differentiation means moving to a superior position on a ‘quality’ dimension that all customers value. But a higher quality firm does not necessarily have a competitive advantage in creating economic value, because customers may differ in their willingness to pay for quality, and because the production of higher quality products generally requires higher variable costs like higher quality inputs or higher skilled labor. As Waterson’s (1989: 2) definition specifies, vertical differentiation means improving some characteristics without worsening any others, and those improvements must be costly, or else no firm would ever produce a low quality product. So, as in the classic model of Mussa and Rosen (1978), we assume that quality improvement raises both customers’ willingness to pay and the firm’s variable cost.³ The firm that is closest to the optimal trade-off between quality and cost has a competitive advantage.

For example, consider the current competition between Wal-Mart and Target in U.S. discount retailing. While there are many differences between the two firms (e.g., their store locations, distribution capabilities, technology infrastructure, and management systems), one important difference is that Target has positioned itself more upscale than Wal-Mart in terms of both product quality and overall shopping experience. This

²Another reason to study both models is that the question of whether there is a significant difference between horizontal and vertical differentiation is not yet settled. Some economists believe they are fundamentally alike, because they can be modeled in mathematically similar ways (Anglin, 1992; Cremer & Thissen, 1991). Yet there are clear differences between them in cases where firms can enter (Shaked & Sutton, 1983) and where some customers choose not to buy at all (Wauthy, 2010).

³Certainly, it is also possible for a quality improvement to involve fixed costs as well (e.g., in the economics literature on research and development races). For simplicity and tractability, we do not consider this possibility in our model of vertical differentiation, but we acknowledge that including fixed costs of quality improvement might make positioning choices in our vertical model behave somewhat more like those in our horizontal model. Also note that, in our horizontal model, we do allow for fixed-cost investments in efficiency improvement, which has a similar character to quality improvement.
vertical differentiation is reflected in surveys indicating that Target shoppers have higher average incomes than Wal-Mart shoppers (Mui, 2005; Pew Research Center for the People and the Press, 2005). But which firm is closest to the optimal trade-off between quality and cost? The answer may depend upon market conditions. As the average income of consumers rises and falls through the business cycle, the optimal trade-off moves along with it. Econometric analysis indicates that Target’s sales performance is pro-cyclical and Wal-Mart’s is counter-cyclical (Basker, 2011), suggesting that Target is closer to the optimal trade-off during economic booms and Wal-Mart is closer to it during recessions.4

In contrast, Waterson’s (1989: 2) definition of horizontal differentiation requires increasing some potentially costly product characteristics while decreasing others. Since these increases and decreases offset each other, there is no reason to expect variable cost to systematically vary with horizontal position, so we maintain the classic assumption that it does not. However, there is a strong reason to expect that horizontal repositioning can substantially affect fixed costs. In the case of a geographical move, there is a one-time cost of physically relocating the firm’s operations and informing customers about the move. Likewise, repositioning in a product-attribute space may require that recipes be reformulated, equipment be retooled, operational procedures be changed, marketing materials be altered, and sourcing techniques or suppliers be switched, all of which changes would entail one-time fixed costs. To our knowledge, our model is the first to recognize the possibility that horizontal repositioning may be costly. We also extend the horizontal model by allowing firms to invest in another universally valuable characteristic, which we model as production efficiency, and examine how this affects their positioning choices.

Industry level: how are industries structured using differentiation?

A firm’s positioning decisions affect not only its own profit but also that of its rivals. This externality is positive when a repositioning restrains rivalry, but negative when a repositioning enhances competitive advantage. In the horizontal model, rivalry restraint and competitive advantage both have positive externalities: moving away from a rival restrains rivalry, which raises the rival’s profit, and moving away from locations preferred by a majority of the market gives the rival a competitive advantage over the differentiating firm, which also raises the rival’s profit. An efficiency improvement by one firm has no direct impact on rivalry, but it gives that firm a competitive advantage, thereby creating a negative externality. In the vertical model, improving quality raises customers’ willingness to pay but also raises production costs. When product quality is low, the former effect outweighs the latter, in which case competitive advantage increases, creating a ‘differentiation advantage,’ which has a negative externality for rivals. When product quality is high, further quality improvement is a competitive disadvantage, which has a positive externality for rivals. In either case, any quality change (either upward or downward) that moves a firm away from its rival restrains rivalry, creating a positive externality.

Unlike rivals who make their positioning decisions independently, cooperating ‘smart competitors’ internalize these externalities in two possible ways. One way is for them to split the market into a profitable duopoly by locating themselves farther apart than they would without cooperation. The other option leads to a diametrically opposite market structure in which one firm monopolizes the market, and the other commits itself to such a large competitive disadvantage that it cannot attract any customers. In the vertical model, cooperating firms always use this monopoly option, which they execute by moving one firm so far away from the optimal quality/cost trade-off that it cannot compete. In the horizontal model, the monopoly option is executed by giving one firm an insurmountable efficiency advantage, but this option is only used if customer heterogeneity is sufficiently low or if efficiency improvement is cheap relative to horizontal differentiation; otherwise, the duopoly option is used. As we discuss later, due to antitrust scrutiny, the duopoly option may often be more feasible.

Intriguingly, in the horizontal model, if customer heterogeneity is low or efficiency improvement is cheap relative to horizontal differentiation, monopoly is a possible equilibrium even if

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4Their stock betas also reflect a difference in cyclicalities. Target’s beta of 0.9 indicates a close connection to overall stock market movements, whereas Wal-Mart’s beta of 0.32 indicates a much weaker connection (Standard & Poor’s, 2011).
firms make their positioning decisions competitively, and there is a broad range of conditions in which both monopoly and duopoly are possible equilibria; the foresight of managers is critical in such cases, since these outcomes have very different implications for profitability. Finally, one might expect customer heterogeneity to reduce the value of cooperative industry structuring, since firms would have strong incentives to differentiate from each other on their own. Yet our horizontal model shows that greater customer heterogeneity enhances the positive externalities from horizontal differentiation, actually raising the benefits of cooperation.

**Firm level: how does differentiation affect profit?**

We study product differentiation at the firm level by considering how the unilateral repositioning of a focal firm, taking its rival’s position as given, affects each firm’s performance. This analysis is a critical building block in our industry-level analysis and also offers valuable insights of its own about big strategic questions. When does differentiation restrain rivalry, and when does it confer competitive advantage? Does the answer depend on the type of differentiation? Does it depend on characteristics of the product market? What constitutes a ‘differentiation advantage’? Using both the horizontal and vertical models, we provide a formal mathematical decomposition of how differentiation affects profitability via rivalry restraint and competitive advantage. Comparing this decomposition between the two models highlights some significant strategic, as opposed to economic, differences between horizontal and vertical differentiation. Lastly, we analyze how product differentiation affects margin and market share, leading to testable implications.

The next two sections of the paper each present a different model—one for horizontal differentiation and one for vertical differentiation. For each model, we first take firms’ positioning as given in order to determine the resulting competitive pricing equilibrium. These results are used to determine how differentiation affects a firm’s profitability. We then consider how the firms should rationally select (either competitively or cooperatively) their positioning at a prior stage, before competing on price. Testable propositions are derived throughout. The final section considers the implications of our results for research, practice, and pedagogy. Technical derivations are largely relegated to the Appendix.

**MODEL 1: HORIZONTAL DIFFERENTIATION**

To model horizontal differentiation, we use a variation on Hotelling’s (1929) classic ‘linear market’ model, as corrected by D’Aspremont and colleagues (1979). Consumers are uniformly distributed on a line segment from $-\sigma$ to $\sigma$, where $\sigma > 0$ captures the degree of customer heterogeneity. This horizontal dimension can be geographic, as in a restaurant’s physical location, or a space of preferences, as in the sweetness of a soft drink, where some consumers prefer less sweetness while others prefer more. A consumer prefers that a product be as close as possible to his or her location on the horizontal dimension, as we explain below. The uniform distribution has a density of $(2\sigma)^{-1}$, so total market size is normalized to 1.

Two firms, designated $H$ for high and $L$ for low, serve this linear market from horizontal positions $h_H$ and $h_L$, respectively, where $h_H \geq h_L$. Each consumer must purchase one and only one indivisible unit of the product, regardless of price, but may purchase the unit from either firm. $H$ and $L$ charge prices $p_H$ and $p_L$, respectively, and cannot price discriminate among consumers. In the final stage of the model, the two firms compete on price, choosing prices noncooperatively to satisfy the requirements of Nash equilibrium. In addition to paying the price of the product purchased, a consumer at location $X$ on the line segment who patronizes firm $i \in \{H,L\}$ pays a transportation cost of $\alpha (X - h_i)^2$ where $\alpha > 0$. If the horizontal dimension is geographic, then the transportation cost can be interpreted literally. If the horizontal dimension is a space of consumer

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5 An implication of this assumption, which is common to models of this kind, is that consumers’ reservation prices for the good cannot be too low. Prior research indicates that this assumption is relatively innocuous in the context of a model like ours. Economides (1984) analyzes spatial competition with ‘low’ reservation prices. He finds that they generally encourage firms to move farther apart to achieve a measure of local monopoly power. This is precisely the same effect that arises from a convex cost of transportation, as in our model and the corrected Hotelling model from D’Aspremont et al. (1979).
preferences, then the transportation cost represents the disutility the consumer experiences as a result of consuming a product that does not perfectly match his or her needs. So, a consumer at location \( X \) would incur a total cost for the product of firm \( i \in \{H,L\} \) of \( p_i + \alpha (X - h_i)^2 \). Each consumer purchases whichever firm’s product has a lower total cost.\(^6\)

All of the assumptions above are consistent with D’Aspremont and colleagues (1979), but we have generalized their model by including the customer heterogeneity parameter \( \sigma \), which will prove quite important in the analysis to follow. In addition, we deviate from their model in two other ways: first, we allow firms to improve the efficiency of their production and thereby improve their value proposition to all consumers, which allows us to explore how differentiation interacts with general competitive strength;\(^7\) second, we provide a novel analysis of the potential costs of horizontal differentiation, which would include the physical relocation of facilities, reformulation of recipes, and rebranding, as well as the risk that the repositioning may fail, which should be regarded as a cost at the time of the repositioning decision.

Specifically, we assume that in the first stage of the model, each firm chooses how much to invest in horizontal differentiation and in improving production efficiency. On the horizontal dimension, we assume that both firms start at the origin (i.e., \( h_H = h_L = 0 \)), and that any horizontal movement by \( H \) must be in the positive direction, and that any horizontal movement by \( L \) must be in the negative direction.\(^8\) We further assume that the cost of horizontal differentiation is quadratic in the distance moved, so that \( H \) would pay \( \beta_h h_H^2 \) to move horizontally from the origin to position \( h_H \), and that \( L \) would pay \( \beta_h h_L^2 \) to move horizontally from the origin to position \( h_L \).\(^9\) This assumption has the natural, intuitive interpretation that the marginal cost of horizontal differentiation increases as the firm moves into regions of geographic or consumer-preference space that are more remote and unfamiliar. \( H \) and \( L \) have efficiency levels of \( e_H \geq 0 \) and \( e_L \geq 0 \) respectively, which represent reductions in per-unit production costs relative to a base level of \( \gamma \). Each firm \( i \) must invest \( \beta_e e_i^2 \) to reduce its per-unit costs of production to \( c_i = \gamma - e_i \). This assumption intuitively implies that there are diminishing marginal returns to investments in efficiency improvement.

Our model has two stages. In the first stage (investment subgame), both firms simultaneously choose their investments in horizontal differentiation and efficiency improvement (\( h_i \) and \( e_i \)). In the competitive version of the investment subgame, each firm maximizes its own profit noncooperatively, whereas in the cooperative version, the firms maximize total industry profit, in anticipation of later competing on price in a second stage. In that second stage (pricing subgame), each firm simultaneously chooses its price noncooperatively to maximize its own profit. For technical reasons, we make the realistic assumption that there is a strictly positive smallest increment \( \varepsilon > 0 \) by which prices can differ (e.g., a penny) but that its size is trivially small, so that it may be ignored in our calculations. One can drop this assumption without affecting any of our results, but at the cost of some technical complications.\(^10\)

The model is solved by backward induction: the second stage pricing equilibrium is derived first, taking the firms’ positioning (\( h_i \) and \( e_i \)) as given. Then, those second-stage results are used to establish rational expectations for the first-stage positioning decisions.

### Pricing subgame: firm-level analysis

There are three mutually exclusive cases for the pricing subgame. The first case occurs if the firms

\[^6\] We preclude a firm from offering multiple products along the horizontal dimension in order to cleanly separate differentiation from diversification, and in order to focus on the most strategically relevant forms of differentiation, that is, those that arise from inherent traits of the firm and that competitors therefore cannot easily imitate by merely introducing more products.

\[^7\] The efficiency dimension also links our model to the value-based framework (Adner and Zemsky, 2006; Brandenburger and Stuart, 1996, 2007; MacDonald and Ryall, 2004), since improvements in efficiency improve a firm’s value proposition to all consumers.

\[^8\] If \( H \) moved negatively and \( L \) positively, we just relabel them, and both moving in the same direction cannot be an equilibrium.

\[^9\] Assuming quadratic costs makes the analysis more tractable, but in principle, any convex, increasing function will produce qualitatively similar results. This is equally true in the case of investments in efficiency improvements, as described later.

\[^10\] If prices are unconstrained in this way, we must resort to mixed strategies in the version of the pricing subgame where one firm has a sufficiently strong net competitive advantage to capture the entire market. The profits of each firm in the mixed strategy equilibrium are the same as in the equilibrium where prices have a minimum increment.
are horizontally differentiated and neither firm has a sufficiently large efficiency advantage to capture the entire market, in which case there is a single interior location \( X \in (−\sigma, \sigma) \) where the consumer is indifferent between \( H \) and \( L \). The second case occurs if the firms are not horizontally differentiated \((h_H = h_L = 0)\) and have identical efficiency levels \((e_H = e_L)\). In this case, both firms price at marginal cost, split the market evenly, and earn zero profits, just as in undifferentiated Bertrand competition. The third case occurs if one firm has a large enough efficiency advantage to capture the entire market \((e_H >> e_L \text{ or } e_H << e_L)\). In that case, the weaker firm prices at marginal cost and earns zero profit, while the stronger firm sets the highest price possible consistent with capturing the entire market.

All three cases are relevant for the first-stage investment subgame, because the firms take into account every possible equilibrium of the pricing subgame when making their positioning decisions. However, for the remainder of this subsection, we consider how changes in one firm’s positioning affect the margin, market share, and profits of both firms on the assumption that neither firm has a large enough competitive advantage to capture the entire market. The Appendix shows that this requires:

\[
\sigma > \sigma_{\text{min}} = \frac{|a_H|}{6\alpha(h_H - h_L)} = \frac{|a_L|}{6\alpha(h_H - h_L)} \tag{1}
\]

where \( H \)'s competitive advantage \( a_H \) (the inverse of \( L \)'s competitive advantage \( a_L \)) is its transportation-cost advantage, averaged across all customers, plus its efficiency advantage:

\[
a_H = -a_L = (e_H - e_L) + \alpha\left(h_L^2 - h_H^2\right) \tag{2}
\]

This definition of competitive advantage is just the difference between the economic value that each firm creates, where ‘economic value’ is the average difference, across all customers, between the customer’s willingness to pay for the product and the firm’s cost to provide it (i.e., ‘\( V - C \)’ in value-based terminology).

Likewise, we define the degree of rivalry restraint in the market as \( r = 2\sigma\alpha(h_H - h_L) \), which intuitively combines all three factors that affect the amount of market share that a firm can attract from its rival by reducing price: (1) the total width of the market, \( 2\sigma \), (2) the cost of transporting the product, \( \alpha \), and (3) the distance between the firms, \( h_H - h_L \). Since each of these factors reinforces the others, it makes sense that they are combined multiplicatively. Moreover, as the Appendix shows, this measure of rivalry restraint equals the average of the two firms’ margins in equilibrium, which is intuitive given that the purpose of rivalry restraint is to raise industrywide margins.

Equation (1) can then be rewritten as \(|a| < 3r\), which means that in order for both firms to have strictly positive market shares, neither firm’s competitive advantage can be too large relative to the market’s degree of rivalry restraint, or else the advantaged firm would monopolize the market. The Appendix derives the firms’ equilibrium prices, market shares, and gross profits under this boundary condition. For simplicity, we consider changes to \( H \)'s positioning, taking \( L \)'s positioning as given.\(^{11}\)

### Main effects

Since \( \partial r / \partial h_H = 2\sigma\alpha > 0 \) and \( \partial a_H / \partial h_H = -2\alpha h_H \leq 0 \), horizontal differentiation increases rivalry restraint for the industry but decreases competitive advantage for the firm. In the Appendix, we decompose the profitability of horizontal differentiation into its effect on rivalry restraint and competitive advantage. This decomposition shows formally that horizontal differentiation moves the firm away from its competitor, thereby reducing the intensity of price rivalry, but also moves the firm away from the majority of consumers; this increases transportation costs to most of the market, which disadvantages the firm relative to its rival. The Appendix shows that the impact of this disadvantage on profit is negligible when horizontal differentiation is low, but it strengthens as horizontal differentiation increases, and eventually outweighs the beneficial effects of restraining rivalry. Consequently, the overall effect of horizontal differentiation on profit has a curvilinear, inverted-U shape.

**Proposition 1.1:** Horizontal differentiation increases rivalry restraint, which raises profit, and decreases competitive advantage, which reduces profit. The reduction in competitive disadvantage is nil when firms are

\(^{11}\)The effect of an increase in \( h_H \) is identical to the effect of a decrease in \( h_L \).
not horizontally differentiated and increases as horizontal differentiation increases, eventually reaching a point where it outweighs the increase in rivalry-restraint. Overall, horizontal differentiation has a curvilinear, inverted-U shaped effect on profit.

Another way to break down the profitability of horizontal differentiation is to examine margin and market share. As the Appendix shows, horizontal differentiation increases margin, but it has two distinct effects on market share: first, horizontal differentiation raises transportation costs to most customers and thereby reduces a firm’s market share; second, by distancing the firm from its rival, horizontal differentiation reduces the impact of any efficiency difference between the two firms. While the first effect is always negative, the second effect is negative if the firm has an efficiency advantage and positive if the firm has an efficiency disadvantage. The second effect attenuates with horizontal differentiation. Therefore, horizontal differentiation reduces market share, except if a firm has an efficiency disadvantage and is located close to its more efficient rival; then, horizontal differentiation boosts market share by blunting the impact of the rival’s efficiency advantage.

Proposition 1.2: Horizontal differentiation has a positive effect on margin, a negative effect on market share for a firm with an efficiency advantage, and a curvilinear inverted-U shaped effect on market share for a firm with an efficiency disadvantage.

Interaction effect with customer heterogeneity

The magnitudes of the effects in the preceding propositions depend upon the degree of customer heterogeneity. As shown in the Appendix, customer heterogeneity reinforces horizontal differentiation’s positive effect on margin, but diminishes its impact on market share, regardless of whether that effect is positive or negative. The reason is the same in each case: in a market with more diffuse customer tastes, firms benefit more by moving apart from each other to cater to market niches and benefit less from efficiency advantages in attracting distant consumers. So, the net impact of customer heterogeneity on the profitability of horizontal differentiation is positive, and it shifts the peak of the inverted-U-shaped effect of horizontal differentiation on profit away from the origin.

Proposition 1.3: As horizontal customer heterogeneity increases, horizontal differentiation has a stronger effect on margin and a weaker effect on market share, and the overall profitability of horizontal differentiation increases. So, there is a broader range of conditions in which further horizontal differentiation would be profitable, and a narrower range of conditions in which further horizontal differentiation would be unprofitable.

If horizontal customer heterogeneity approaches infinity, horizontal differentiation does not affect market share and so monotonically increases profit. Likewise, the effect of horizontal differentiation is monotonic as customer heterogeneity approaches the lower bound in Equation (1), but can be either positive or negative. Profit declines monotonically with horizontal differentiation if $H$ has a net positional advantage (i.e., if $a_H = -a_L > 0$), but increases monotonically if $H$ has a net positional disadvantage (i.e., if $a_H = -a_L < 0$).

Externalities

A unilateral increase in efficiency by one firm decreases the profitability of its rival, a negative externality. This makes sense because efficiency confers a competitive advantage, and one firm’s competitive advantage is its rival’s disadvantage. But increasing a firm’s horizontal differentiation increases the profitability of its rival, due to two positive externalities: first, the rivalry-restraining benefit of horizontal differentiation boosts the profitability of both firms; second, a firm that disadvantages itself by horizontally differentiating away from the majority of customers is simultaneously advantaging its rival.

Proposition 1.4: Horizontal differentiation increases the profit of a rival firm, and efficiency improvement decreases the profit of a rival firm.

Investment subgame: industry-level analysis

We now allow horizontal differentiation and efficiency investments to be determined endogenously
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at a prior stage, before price competition begins. So, we now adjust the firms’ equilibrium gross profits from the second stage (designated with an asterisk) by deducting the cost of positioning investments. The firms’ resulting first-stage objective functions (subscripted $N$ to denote profits ‘net’ of these investment costs) are:

$$\Pi_{HN} = \Pi_{HG}^* - \beta_h h_H^2 - \beta_e e_H^2 \quad \text{and}$$
$$\Pi_{LN} = \Pi_{LG}^* - \beta_h h_L^2 - \beta_e e_L^2 \quad (3)$$

We consider two versions of the first-stage investment subgame: in the competitive version, each firm independently maximizes its own net profit function. In the cooperative version, firms jointly maximize total industrywide net profit, $\Pi_{TN} = \Pi_{HN} + \Pi_{LN}$, in making their investment decisions, yet still compete on price in the second stage.

**Competitive positioning**

There are two types of pure-strategy Nash equilibria in the competitive version of the investment subgame\(^{12}\): in a symmetric equilibrium, the two firms make symmetric investments in the first stage and then split the market evenly in the second stage. In a monopoly equilibrium, one firm invests to gain an insurmountable efficiency advantage that ensures that it will monopolize the market, and the other firm rationally invests nothing in the first stage, because any such investment would be wasted. The Appendix derives these equilibria, and describes how we use numerical analyses to verify that these equilibria are valid and that no other equilibria exist.

Figure 1 provides a sample illustration showing the range of parameter values where each type of equilibrium exist. Three observations can be made from this figure: first, the monopoly equilibrium exists wherever the symmetric equilibrium does not and vice-versa. Second, the symmetric equilibrium does not exist when $\sigma$ is small, $\beta_h$ is large, or $\beta_e$ is small. So, when it is relatively cheap to develop a competitive advantage by investing in efficiency or when customers’ preferences are not heterogeneous enough, monopoly prevails. Then, the firms are in a race to capture the market.

Conversely, if the cost of improving efficiency, $\beta_e$, rises sufficiently, acquiring a monopoly is not feasible. Third, for some parameter values, both the symmetric and monopoly equilibria exist. So, even without ‘irrational blunders’ by either firm, the question of whether a firm will end up as a moderately profitable competitor in a horizontally differentiated market, as a highly profitable monopolist, or as a ‘wallflower’ shut out of the market by a rival is not fully determined by the environment. It follows that a manager’s strategic foresight in shaping outcomes, that is, in making one equilibrium more or less likely, can have a dramatic effect on the profitability of his or her firm.

Suppose that customer heterogeneity is large enough to support the symmetric equilibrium. The Appendix shows that, in this case, greater customer heterogeneity yields greater horizontal differentiation, which allows firms to set higher prices and earn higher net profits.

**Cooperative positioning**

Just as in the competitive positioning scenario, cooperative positioning can also lead to either a symmetric solution or a monopoly solution. For each solution, the Appendix analyzes the associated investment levels, prices, and net profits, along with the conditions under which each solution generates the highest total industry profits. Again, the monopoly solution dominates the symmetric solution when $\sigma$ is small, $\beta_h$ is large, or $\beta_e$ is small, and higher levels of customer heterogeneity yield greater horizontal differentiation, which allows firms to set higher prices, which in turn generates higher net profits. We accordingly strengthen and extend Proposition 1.3 as follows:

**Proposition 1.5:** Regardless of whether firms make positioning decisions competitively or cooperatively, as horizontal customer heterogeneity increases, firms invest more in horizontal differentiation, set higher prices, and earn higher profits.

**Competitive versus cooperative positioning**

Let us restrict our attention to the range of parameter values where both firms have positive market share with both cooperative and competitive positioning. We can then ask which leads to more
horizontal differentiation. Proposition 1.4 says that the externality effect on a rival’s profit is positive for horizontal differentiation but negative for efficiency improvement. If firms make positioning decisions competitively, they do not internalize these externalities and therefore underinvest in horizontal differentiation and overinvest in efficiency improvement relative to the industrywide optimum.

Proposition 1.6: Relative to firms that position themselves competitively, firms that cooperate in their positioning invest more in horizontal differentiation and less in efficiency improvement, and consequently set higher prices and earn higher profits.

We know from Proposition 1.5 that, with both competitive and cooperative positioning, higher customer heterogeneity motivates firms to increase their horizontal differentiation. The Appendix shows that this rate of increase is higher when positioning is cooperative than when it is competitive. The reason is that firms cooperating in their positioning internalize all, not just part, of the rivalry-restraining benefit of horizontal differentiation. So, as customer heterogeneity increases, firms that cooperate in positioning are more motivated to take advantage of the increased opportunity for rivalry restraint by differentiating further.

Proposition 1.7: As customer heterogeneity increases, the horizontal differentiation and profits of firms that cooperate in their positioning increase at a faster rate than if they position themselves competitively.

MODEL 2: VERTICAL DIFFERENTIATION

While customers in Model 1 diverge in their preferences about what constitutes the ideal qualities of a product, customers in Model 2 diverge in their willingness to pay for a quality that all consumers desire. The formal assumptions of Model 2 are as follows. Each consumer’s willingness to pay for a unit increase in quality is represented by the variable $Y$, which is distributed uniformly across consumers from a minimum value of $\mu - \delta$ up to a maximum value of $\mu + \delta$, with density of $(2\delta)^{-1}$ (to normalize the total market size to 1). $\delta \geq 0$ represents the degree of customer heterogeneity. \(^{13}\)

---

13Although it is natural to assume $\mu - \delta > 0$ so that all customers place a positive value on quality, all of our propositions hold if $\mu - \delta < 0$, in which case some customers actually dislike this type of quality.
Further assume that in order for firm \( i \) to increase its quality \( v_i \geq 0 \), the firm must incur higher per-unit marginal costs of \( c_i = \gamma + \omega v_i^2 \) for some \( \omega > 0 \). This quadratic function captures the idea of diminishing returns to quality improvement, that is, as quality improves, the marginal cost of further quality improvements increases at an increasing rate. We make the natural and intuitive assumption that there is a lowest possible quality level (e.g., the quality of a product that costs nothing to produce), and we normalize that minimum quality level to be zero. We continue to denote the two firms as \( H \) and \( L \), and without loss of generality, we assume that \( v_H \geq v_L \), so \( H \) is the high-end firm and \( L \) is the low-end firm. As with Model 1, we assume that firms choose these vertical positions in a first stage, and then compete on price in the second stage. We again solve by backward induction, using profits anticipated for the second stage to inform the positioning decisions in the first stage.

Consistent with Model 1, we follow the value-based strategy definition of competitive advantage (Brandenburger and Stuart, 1996) as the difference between the amount of economic value that the two firms create, where ‘economic value’ is the average difference, across all customers, between the customer’s willingness to pay for the product and the company’s cost to provide it:

\[
\begin{align*}
\text{Economic Value} &= \mu (v_H - v_L) - \omega (v_H^2 - v_L^2) \\
&= \mu (v_H - v_L) - \omega \left[ \frac{1}{2} (v_H - v_L)^2 \right] \\
&= \mu (v_H - v_L)\left[ 1 - \omega \frac{(v_H - v_L)}{2\mu} \right] \\
&= \mu (v_H - v_L)\left[ 1 - \omega \frac{g}{2\mu} \right] \\
&= \mu (v_H - v_L)\left[ 1 - \omega \frac{|wi|}{2\mu} \right]
\end{align*}
\]  

Equation (5) means that if there is not enough customer heterogeneity relative to the average value created, across the entire market, by a unit increase in industrywide quality (i.e., the difference between the average willingness to pay, across the entire market, for an extra unit of quality, \( \mu \), and the marginal cost of both firms of offering that extra unit of quality), then one firm will monopolize the market. Equation (5) can also be rewritten as \(|wi| < 3g\), which means that in order for both firms to have positive market shares, neither firm’s competitive advantage can be too large relative to the market’s degree of rivalry restraint, or else the advantaged firm would monopolize the market.

**Pricing subgame: firm-level analysis**

As we did in Model 1, let us for the moment maintain the assumption, per Equation (5), that both firms have positive market shares. The Appendix derives the equilibrium prices, market shares, and profits under this assumption. Unlike Model 1, Model 2 is inherently asymmetric, because if there were no price difference, all customers would buy from the high-end firm, so we must analyze each firm separately.

**Main effects**

In the Appendix, the effect of improving quality on a firm’s margin is decomposed into a competitive advantage effect and a rivalry restraint effect. The competitive advantage effect captures the trade-off between consumers’ higher willingness to pay for a product and the higher cost of producing it. Because of this trade-off, the competitive advantage effect has a curvilinear inverted-U shaped influence on margin. When quality is low, the benefit of boosting customer willingness to pay outweighs the cost of producing higher quality, thereby increasing competitive advantage, which in turn increases margin. However, the marginal cost of production rises at a faster rate than willingness to pay. Eventually, the higher costs of production reduce margin. By contrast, the rivalry restraint effect is monotonic but differs for each firm. Quality improvement by \( H \) moves its product away from \( L \), thereby increasing rivalry restraint, which in turn raises \( H \)’s margin. Conversely, quality improvement by \( L \) moves its product closer to \( H \), thereby decreasing rivalry restraint, which in turn reduces \( L \)’s margin.

Adding the rivalry restraint effect to the competitive advantage effect shifts the peak in quality’s
curvilinear inverted-U shaped effect on margin. The peak shifts upward to a higher quality level for \( H \) and downward to a lower quality level for \( L \). The size of the shift is proportional to customer heterogeneity. For sufficiently high customer heterogeneity, the level of quality at which \( L \)’s margin peaks may be negative, which is not feasible, so the total effect of quality on \( L \)’s margin would be strictly negative in that case.

**Proposition 2.1:** Quality improvement affects a firm’s margin via competitive advantage and via rivalry restraint. The competitive advantage effect of quality improvement has a curvilinear influence on margin. The rivalry restraint effect of quality on margin is positive if the repositioning moves the firm closer to its rival and negative if the repositioning moves the firm farther from its rival. The total effect of quality on margin is curvilinear for a high-end firm. The total effect of quality on margin for a low-end firm is curvilinear if customer heterogeneity is low, and negative if customer heterogeneity is high.

The Appendix shows that if \( L \) increases its quality, it will have a larger market share, but if \( H \) increases its quality, it will have a smaller market share. In other words, either firm gains market share as it moves toward the competing firm’s quality level, that is, as \( L \) increases quality, or as \( H \) decreases quality. This is analogous to the result from Model 1, Proposition 1.2, that horizontal differentiation reduces a firm’s market share unless the firm is at a severe efficiency disadvantage.

**Proposition 2.2:** Repositioning toward the quality level of a rival increases a firm’s market share.

The Appendix provides a similar decomposition for quality improvement’s effect on profitability. As with margin, the competitive advantage effect has an inverted-U shaped curvilinear influence on profit, whereas the rivalry-restraint effect has a positive influence on profit for \( H \) and a negative influence on profit for \( L \). The Appendix shows that the total effect of quality on profit is qualitatively similar to the total effect of quality on margin, and for the same reason. For \( H \), the total effect is always curvilinear; for \( L \), it is curvilinear if customer heterogeneity is sufficiently low, but negative otherwise.

**Proposition 2.3:** Quality improvement affects a firm’s profit via competitive advantage and via rivalry restraint. The competitive advantage effect of quality improvement has a curvilinear influence on profit. The rivalry restraint effect of quality on profit is positive if the repositioning moves the firm closer to its rival and negative if the repositioning moves the firm farther from its rival. The total effect of quality on profit is curvilinear for a high-end firm. The total effect of quality on profit for a low-end firm is curvilinear if customer heterogeneity is low, and negative if customer heterogeneity is high.

**Interaction with customer heterogeneity**

The effects in the three preceding propositions are affected by customer heterogeneity in willingness to pay for quality. The Appendix shows that customer heterogeneity improves the effect of raising quality on \( H \)’s margin and market share and worsens the effect on \( L \)’s margin and market share. This makes sense because increased heterogeneity leaves fewer customers to compete for in the middle, so firms gain more monopoly power by spreading apart.

**Proposition 2.4:** As customers become more heterogeneous in willingness to pay for quality, the effects of quality improvement on margin and market share become more negative for a low-end firm and more positive for a high-end firm.

The Appendix shows that as customer heterogeneity increases, the competitive advantage effect of quality improvement on profits is initially ambiguous but eventually approaches zero. By contrast, greater customer heterogeneity makes the rivalry restraint effect of quality improvement more positive for \( H \) and more negative for \( L \), again because the firms gain more monopoly power by diverging. It follows that if customer heterogeneity is sufficiently large, any further increase in heterogeneity makes the total effect of quality improvement on profitability more positive for \( H \) and more negative for \( L \).
Proposition 2.5: Customer heterogeneity makes the rivalry restraint effect of quality improvement more positive for a high-end firm and more negative for a low-end firm. If customers are sufficiently heterogeneous in their willingness to pay for quality, any further increase in heterogeneity makes the total effect of quality improvement on profitability more positive for a high-end firm and more negative for a low-end firm.

Externalities

As shown in the Appendix, quality improvement by $H$ restrains rivalry by moving it away from $L$, which increases $L$’s profit. Conversely, quality improvement by $L$ increases rivalry by moving it toward $H$, which decreases $H$’s profit. The intuition is the same as in Model 1’s Proposition 1.4.

Proposition 2.6: Repositioning a firm’s quality away from a rival (upward for a high-end firm retreatting from a low-end rival, or downward for a low-end firm retreatting from a high-end rival) raises the rival’s profit.

Investment subgame: industry-level analysis

Competitive positioning

We now allow quality positioning to be determined endogenously at a prior stage, before price competition begins. As shown in the Appendix, if these first-stage positioning choices are made competitively, then both firms always have strictly positive market shares in the second stage, and greater customer heterogeneity in willingness to pay for quality causes the firms to diverge more in their quality levels, with $H$ increasing its quality and $L$ decreasing its quality. The reason is that greater customer heterogeneity means fewer customers in the middle to compete for, which increases the gain in monopoly power when the firms move apart. For the same reason, both firms’ margins rise as customer heterogeneity increases. Once customer heterogeneity is so large that $L$ chooses the minimum quality of zero, $v_L = 0$, further increases in customer heterogeneity cause $H$ to move upscale faster than it otherwise would. Why? If $H$ continued moving upscale at the same rate even though $L$ was no longer able to move downward, then the amount of rivalry-restraint generated by further increases in customer heterogeneity would roughly be cut in half. In order to mitigate that loss of rivalry restraint, $H$ partially compensates for $L$’s immobility by accelerating its upward movement. But why is it only partially compensated? As $H$ moves ever higher upscale, its distance from the optimal quality/cost trade-off increases while $L$’s distance from this optimum remains constant. This asymmetry imposes a competitive disadvantage on $H$. So, $H$ chooses a rate of upward movement that balances its gain from rivalry restraint against its loss of competitive advantage. Even so, $H$ suffers an increasing competitive disadvantage and loses market share to $L$.

Proposition 2.7: If rival firms make positioning investments competitively, then increasing customer heterogeneity in willingness to pay for quality motivates high-end and low-end firms to diverge from each other in their quality levels, and thereby earn both higher margins and profits. When customers’ willingness to pay for quality is so heterogeneous that the low-end firm chooses the minimum possible level of quality, any further increase in heterogeneity causes a low-end firm to gain market share and a high-end firm to lose market share.

Cooperative positioning

If positioning choices are made cooperatively, then the firms will position $L$ to monopolize the market. As shown in the Appendix, this goal is accomplished by having $L$ choose a positive finite quality level, and by having $H$ choose an arbitrarily high quality level. As a result, $H$’s market share disappears, and $L$’s profit, and thus the firms’ joint profits, can grow arbitrarily large.

Competitive versus cooperative positioning

Obviously, because the firms’ joint profits and the difference between their quality levels are arbitrarily large in the cooperative case, these will always be greater than with competitive positioning.

Proposition 2.8: Relative to firms that position themselves competitively, firms that cooperate in vertical positioning choose a
greater disparity in quality and earn higher profits.

As with Model 1, this difference is due to the internalization of externalities. The divergence of the firms into two separate quality-based vertical niches benefits both firms by reducing the intensity of their price rivalry, but each firm must bear a private loss in order to achieve this shared benefit. For $H$, this private loss comes in the form of a higher marginal cost of production, while for $L$, it comes in the form of decreased customer willingness to pay. When firms position themselves competitively rather than cooperatively, each firm is only willing to bear enough private loss to maximize its own individual profit, which is insufficient to maximize industry profits. Moreover, these private losses are not symmetric. $H$’s production costs increase at an increasing rate while $L$’s customer willingness to pay decreases linearly. Eventually, $H$ winds up bearing the lion’s share of these losses, especially after $L$’s quality level reaches its minimum possible value of $v_L^* = 0$. Without cooperative positioning, $H$ is naturally unwilling to make sacrifices for which its rival is the primary beneficiary.

DISCUSSION

In this paper, we take the idea of industry structuring seriously. Although much is known about industry structure (noun), much less is known about industry structuring (verb). In other words, we know more about how industry structure affects economic outcomes than about how rival firms can, either individually or collectively, shape their industry’s structure for their benefit. This paper explores how firms can structure their industry using product differentiation, which may allow firms to approach Stigler’s (1964: 46) ideal of assigning each buyer to a single seller without violating antitrust law. We examine both the horizontal and vertical versions of a two-stage model in which rival firms make product differentiation decisions either competitively or cooperatively in anticipation of subsequent price competition.

Our models indicate that if firms cooperate in their positioning, they internalize positive externalities from rivalry-restraining differentiation and negative externalities from creating a ‘differentiation advantage.’ So, firms adopt more distinctive positions and earn higher profits when they cooperate like this than when they do not. Our results also indicate that customer heterogeneity raises the incentive to be ‘smart competitors’ in this way. We show that cooperation can be accomplished either by setting up a profitable duopoly with firms occupying distinct niches or by setting up one firm to monopolize the market. Monopoly may be harder to implement legally, because it would in principle require the monopolizing firm to make side transfers to the excluded firm. One way of doing this may be through merger, for example, when Sirius and XM merged to create a monopoly in the U.S. satellite radio market in 2007. Yet, as noted, such mergers would frequently fall afoul of antitrust authorities. If so, our vertical model suggests that firms might still cooperate profitably in positioning. One alternative would be to choose a less extreme level of rivalry restraint than monopoly, whereby both the high-end and the low-end firm retained positive market share. Indeed, this duopoly alternative may be the most economical outcome if, as assumed in our horizontal model, there are fixed costs of differentiation.14 However, our horizontal model shows that if side payments are impossible, there may be limits to industry structuring through differentiation. The reason is that monopoly is more profitable than duopoly precisely when monopoly is also the equilibrium from making positioning decisions competitively. Firms would accordingly find it more profitable to race to become the monopolist by investing in efficiency than to set up a duopoly, which would generate lower profit for both firms than a monopoly would for one firm. Exploring the impact of such constraints is an exciting topic for future research.

Another goal of this paper is to provide the analytical precision needed to dissect the causal mechanisms that motivate product differentiation. Product differentiation may affect profit by increasing the competitiveness of a firm and by decreasing the competitiveness of its industry, and these two mechanisms are not independent of each other.14 By contrast, in our model of vertical differentiation, the absence of such costs made it possible to position the high-end firm infinitely far away for free because that distant positioning would leave it with zero market share, and therefore zero variable cost.
Our decomposition reveals the following: (1) horizontally and vertically differentiating away from a rival restrains rivalry; (2) if firms have similar efficiency, horizontal differentiation reduces competitive advantage by making a firm’s product less appealing to the majority of the market, whereas vertical differentiation increases competitive advantage (to create a differentiation advantage) if and only if the firm moves closer to the optimal trade-off between quality and marginal cost; and (3) horizontal differentiation normally reduces market share but may increase market share if a firm has an efficiency disadvantage by establishing some breathing room from a stronger rival.

In that connection, the strategy literature pays little heed to the fact that product differentiation can lead to a competitive disadvantage by reducing the firm’s value creation across the market as a whole. Research, pedagogy, and practice should recognize that horizontal differentiation usually reduces competitive advantage, and that vertical differentiation may do so as well if taken too far—beyond the optimal quality/cost trade-off. Theory and pedagogy should also recognize that differentiation simultaneously affects rivalry restraint and competitive advantage, which may have counteracting effects on profit.

Our models offer a new interpretation of the oft-repeated critique in the practitioner literature that firms do not differentiate enough. To wit, Kim and Mauborgne (2005) claim that managers neglect opportunities to create ‘new market space’ by differentiating into ‘blue oceans.’ Hamel and Prahalad (1994) see firms as neglecting various ‘industry transformation’ possibilities, including introducing novel product characteristics that would differentiate the firms in fundamentally new ways. Such insufficient differentiation could result from the personal failings of managers or the organizational weaknesses of firms. Hamel and Prahalad (1994) also claim that managers are too focused on making their products more competitive on existing standards or metrics rather than on inventing new standards or metrics, and Porter (1980: 42) blames a combination of bureaucratic inertia and indecisive or inconsistent managers. Treacy and Wiersema (1995: 44) see firms as mistaking mere improvement for real competitive superiority and as being reluctant to make ‘hard choices’ like abandoning customers who will not pay more for differentiation, while Kim and Mauborgne (2005) suggest that managers lack the skills and conceptual frameworks to differentiate their businesses in truly innovative ways. There is likely some truth in each of these claims. We add another possibility to this list: to be ‘smart competitors’ à la Yoffie requires that managers agree to internalize the effects of their positioning on each other. By contrast, an industry composed of fully rational but less cooperative managers will resemble the less differentiated competitive equilibria of our models.

Some caveats about our results are warranted. First, they were derived from formal modeling, which has the virtue of transparency, internal consistency, and a replicable ‘audit trail’ but lacks the ‘complexity of a real-life encounter’ (Adner et al., 2009: 205). Any attempt at either practical or empirical application of our results should be made with caution and appropriate adjustments to the specific situation.

We highlight a few of the more obvious limitations of our analysis here, which, if relaxed, could represent promising avenues for future research. First, to focus on differentiation apart from diversification, we assumed that each firm has only one competitive position. If firms had the option of diversifying in our spaces of consumer preferences, then competitive positioning would likely lead them to ‘overdiversify’ relative to the cooperative optimum, since they would only internalize the cannibalization of their own sales, not sales of rivals. We also assumed that firms have symmetric positioning opportunities and costs. Preexisting asymmetries in capabilities and positioning may lead to asymmetries in outcomes, although our model showed that such preexisting asymmetries are not necessary for an asymmetric outcome. We also assumed a production technology without scale economies. With scale economies, market share would be more valuable, so competition to gain share via competitive advantage would be more intense. Another interesting possibility that we do not consider are markets where scarcity is part of what makes the product desirable (e.g., Swatches). Our model also restricted entry. Without this assumption, incumbents would take into account how their positioning decisions affected entry. Finally, we made simplifying assumptions about the distribution of consumer tastes (uniform) and reservation prices (infinite), and changing these assumptions (e.g., a bimodal distribution of customers) may change the optimal competitive positioning.
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REFERENCES


APPENDIX

Model 1: Pricing subgame

If the firms are horizontally differentiated and neither firm has a large enough competitive
advantage to capture the entire market, there must be a single marginal consumer located at an interior point \( \tilde{X} \in (-\sigma, \sigma) \) who is indifferent between firms \( H \) and \( L \). This occurs where:

\[
p_H + \alpha \left( \tilde{X} - h_H \right)^2 = p_L + \alpha \left( \tilde{X} - h_L \right)^2 \tag{6}
\]

Solving this equation for \( \tilde{X} \) yields:

\[
\tilde{X} = \frac{h_H + h_L}{2} + \frac{p_H - p_L}{2\alpha (h_H - h_L)} \tag{7}
\]

This implies demand functions and gross (i.e., before deducting investment costs) profit functions of:

\[
q_H = \frac{1}{2} - \frac{h_H + h_L}{4\sigma} - \frac{p_H - p_L}{4\sigma \alpha (h_H - h_L)} \quad \text{and} \quad q_L = 1 - q_H \tag{8}
\]

\[
\Pi_{HG} = q_H (p_H - \gamma + e_H) \quad \text{and} \quad \Pi_{LG} = q_L (p_L - \gamma + e_L) \tag{9}
\]

Differentiating each \( \Pi_{iG} \) with respect to its corresponding \( p_i \), setting both derivatives equal to zero, and solving the resulting equation system yields the second-stage price equilibrium:

\[
p_H^* = (\gamma - e_H) + (1/3) \left[ \alpha (h_H - h_L) (6 - (h_H + h_L)) + (e_H - e_L) \right] = (\gamma - e_H) + 4\sigma \alpha (h_H - h_L) q_H^* \\
p_L^* = (\gamma - e_L) + (1/3) \left[ \alpha (h_H - h_L) (6 + (h_H + h_L)) - (e_H - e_L) \right] = (\gamma - e_L) + 4\sigma \alpha (h_H - h_L) q_L^* \tag{10}
\]

Define the level of rivalry restraint, \( r \), as the average margin of the two firms in equilibrium:

\[
r = (1/2) \left[ (p_H^* - (\gamma - e_H)) + (p_L^* - (\gamma - e_L)) \right]
= 2\sigma \alpha (h_H - h_L) \tag{11}
\]

Substituting the prices from (10) into Equations (8) and (9) yields market shares and gross profits of:

\[
q_H^* = \frac{1}{2} \left( 1 + \frac{a_H}{3r} \right) \quad \text{and} \quad q_L^* = \frac{1}{2} \left( 1 + \frac{a_L}{3r} \right) \tag{12}
\]

\[
\Pi_{HG}^* = \frac{r}{2} \left( 1 + \frac{a_H}{3r} \right)^2 = 2r (q_H^*)^2 \quad \text{and} \quad \Pi_{LG}^* = \frac{r}{2} \left( 1 + \frac{a_L}{3r} \right)^2 = 2r (q_L^*)^2 \tag{13}
\]

This equilibrium is only valid if both firms have positive market shares. Algebraic manipulation of Equation (12) shows that this is true if and only if the boundary condition in Equation (1) is satisfied. If not, one firm has a strong enough advantage to monopolize the market. Without loss of generality, let this firm be \( H \), since the case where \( L \) monopolizes the market is analogous. In the Nash equilibrium for this case, \( q_L = 0 \) and \( q_H = 1 \), and neither firm has an incentive to unilaterally change its price, which occurs only at:

\[
p_H^* = \gamma + a_H - r - \epsilon \quad \text{and} \quad p_L^* = \gamma \tag{14}
\]

where \( \epsilon > 0 \) is the smallest possible increment by which prices can be adjusted. Since \( \epsilon \) was assumed to be trivially small, we ignore it for the remainder of the analysis. If \( L \) lowers its price, then it would suffer negative margin. If \( H \) raises its price, then its profits remain zero. If \( H \) lowers its price, then it will reduce its margin without any compensating increase in market share. To show that \( H \) has no incentive to raise its price, we substitute the prices from Equation (14) into the derivative of \( H \)'s profit function from Equation (9) to yield the condition \( a_H \geq 3r \), which violates the condition in Equation (1). We then have that \( L \) has a gross profit of zero and that \( H \) has a gross profit of \( \Pi_{HG}^* = a_H - r \). Alternatively, if the firms are not horizontally differentiated and neither firm has an efficiency advantage, standard Bertrand logic implies that both price at marginal cost, split the market (as buyers are indifferent between them), and earn no profit.

Returning now to the duopoly equilibrium in Equations (10) through (13), if this equilibrium is valid (i.e., if Equation (1) is true), then efficiency
has positive main effects on market share, margin, and profit:

\[
\frac{\partial q_H^*}{\partial e_H} = (6r)^{-1} > 0 \quad \text{and} \\
\frac{\partial m_H^*}{\partial e_H} = \frac{\partial}{\partial e_H} \left( p_H^* - (\gamma - e_H) \right) = \frac{1}{3} > 0 \quad \text{and} \\
\frac{\partial \Pi_{HG}^*}{\partial e_H} = \frac{2q_H^*}{3} > 0
\]  

The effect of horizontal differentiation on profitability can then be decomposed as follows:

\[
\frac{\partial \Pi_{HG}^*}{\partial h_H} = \left[ \left( \frac{3r + a_H}{9r} \right) \frac{\partial a_H}{\partial h_H} \right] \\
+ \left[ \left( \frac{(3r + a_H)(3r - a_H)}{18r^2} \right) \frac{\partial r}{\partial h_H} \right]
\]  

(15)

The first term in this derivative relates to competitive advantage, while the second term relates to rivalry restraint. By Equation (1), we know that \(-3r < a_H < 3r\), so the fractions in both terms are positive. Since \(\partial a_H/\partial h_H < 0\) and \(\partial r/\partial h_H > 0\), the first term is negative, and the second term is positive. The net effect depends upon which term is larger in magnitude. Equation (17) confirms that horizontal differentiation has a curvilinear, inverted-U shaped effect on profit, that is, positive when \(h_H^*\) is low and negative when \(h_H^*\) is high:

\[
\frac{\partial \Pi_{HG}^*}{\partial h_H} = 4\alpha e_H^* \left( q_L^* - \frac{h_H^*}{3\sigma} \right) \geq 0 \\
\text{iff} \quad h_H^* \leq 3\sigma q_L^*
\]  

(16)

Equations (16) and (17) are the basis for Proposition 1.1.

Differentiating margin establishes the first part of Proposition 1.2:

\[
\frac{\partial m_H^*}{\partial h_H} = \frac{\partial}{\partial h_H} \left( p_H^* - \gamma + e_H \right) = \frac{2\alpha}{3} (3\sigma - h_H) > 0 \\
\text{iff} \quad h_H < 3\sigma
\]  

(18)

This derivative is positive unless the firm has moved farther from its nearest consumer than that consumer is from the opposite endpoint (i.e., unless \(h_H > 3\sigma\)), at which point any further differentiation would require price discounts in order to compensate consumers for incurring extraordinarily high transportation costs. Since the motivation for horizontal differentiation is to increase margins, we limit our attention to the range of parameters where such margin increase is possible, which implies \(h_H \in [0, 3\sigma]\). In fact, neither firm would ever position itself past \(\pm 3\sigma/2\) when positions are chosen competitively.

Differentiating market share establishes the second part of Proposition 1.2:

\[
\frac{\partial q_H^*}{\partial h_H} = -\frac{1}{12\sigma} - \frac{(e_H - e_L)}{12\sigma^2 (h_H - h_L)^2}
\]  

(19)

The first term is always negative, reflecting the firm’s growing distance from the majority of customers. The second term shows how the effect of any efficiency difference on market share attenuates as the firms move apart. This second term may be positive or negative, depending upon whether the firm has an advantage or disadvantage in efficiency. The net effect is negative unless the firm has a disadvantage in efficiency (i.e., \(e_H < e_L\)) and is positioned near its rival’s horizontal location (i.e., \(h_H - h_L\) is small).

Differentiating Equations (18) and (19) with respect to \(\sigma\) yields:

\[
\frac{\partial m_H^*}{\partial \sigma \partial h_H} = \frac{\partial}{\partial \sigma \partial h_H} \left( p_H^* - \gamma + e_H \right) = 2\alpha > 0 \quad \text{and} \\
\frac{\partial q_H^*}{\partial \sigma \partial h_H} = \frac{1}{12\sigma^2} + \frac{(e_H - e_L)}{12\sigma^2 \alpha (h_H - h_L)^2} = -\sigma^{-1} \frac{\partial q_H^*}{\partial h_H}
\]  

(20)

Customer heterogeneity reinforces horizontal differentiation’s positive effect on margin but diminishes its effect on market share by reducing the relevance of both the firm’s distance to the majority of customers and its relative efficiency. The net impact of customer heterogeneity on the profitability of horizontal differentiation is the derivative of Equation (17) with respect to \(\sigma\):

\[
\frac{\partial^2 \Pi_{HG}^*}{\partial \sigma \partial h_H} = \frac{4\alpha}{3\sigma} \left[ h_H (q_H^*)^2 + (3\sigma - h_H) \left( \frac{1}{2} q_H^* + (q_H^*)^2 \right) \right]
\]  

(21)
which is positive if $0 < q_H^* < 1$ and $h_H < 3\sigma$, as we have assumed. So, by the monotone comparative-statics theorem (Topkis, 1998), the peak of the inverted-U-shaped effect of horizontal differentiation on profit shifts outward as $\sigma$ increases. Equations (20) and (21) are the basis for Proposition 1.3.

Equations (12) and (19) show that, as $\sigma$ approaches infinity, the effect of horizontal differentiation on market share disappears, so its effect on profit becomes monotonically positive. If $a_H = -a_L > 0$, then $q_L^*$ approaches 0 as $\sigma$ approaches $\sigma_{\text{min}}$, so the level of horizontal differentiation at which profit starts to decline, $\tilde{h}_H = 3\sigma_q_L^*$, must also approach 0. In that case, profit would decline monotonically over the full range $h_H \in [0, 3\sigma)$. If $a_H = -a_L < 0$, then $q_L^*$ approaches 1 as $\sigma$ approaches $\sigma_{\text{min}}$, so the level of horizontal differentiation at which profit starts to decline, $\tilde{h}_H = 3\sigma_q_L^*$, must approach $3\sigma$. In that case, profit would increase monotonically over the full range $h_H \in [0, 3\sigma)$. If $a_H = a_L = 0$, then $q_L^* = q_H^* = 1/2$ and $\sigma_{\text{min}} = 0$, so the full range $h_H \in [0, 3\sigma)$ would collapse down to the origin as $\sigma$ approaches $\sigma_{\text{min}}$.

To establish Proposition 1.4, note that horizontal differentiation by $H$ affects $L$'s profit as follows:

$$
\frac{\partial \Pi_{LL}^*}{\partial \hat{e}_L} = \frac{-2q_L^*}{3} < 0
$$

and

$$
\frac{\partial \Pi_{LL}^*}{\partial \hat{h}_H} = \left( \frac{3\sigma + a_L}{9\sigma} \right) \frac{\partial a_L}{\partial h_H} + \left( \frac{3\sigma + a_L}{18\sigma^2} \right) \frac{\partial r}{\partial h_H} > 0
$$

(22)

Model 1: Investment subgame

We first consider competitive positioning. Assume that both firms anticipate positive market share. Differentiating each firm’s net profit function in Equation (3) with respect to each of its positioning variables $(\hat{e}_i, \hat{h}_i)$ and setting these derivatives equal to zero yields a system of four equations in four variables, but its solution is intractable, so we conjecture and then verify that one solution to the system is symmetric across firms. Imposing symmetry across firms yields the following solution:

$$
\tilde{h}_H = -\tilde{h}_L = \frac{3\alpha\sigma}{2(\alpha + 3\beta_h)} \quad \text{and}
\tilde{e}_H = \tilde{e}_L = \frac{1}{6\beta_e}
$$

(23)

where the tilde indicates competitive equilibrium. The firms’ prices and net profits are then

$$
\tilde{p}_H^{ss} = \tilde{p}_L^{ss} = \gamma + \frac{6\alpha^2\sigma^2}{\alpha + 3\beta_h}
$$

and

$$
\tilde{\Pi}_{HN}^{ss} = \tilde{\Pi}_{LN}^{ss} = \frac{1}{36} \left( \frac{27\alpha^2\sigma^2}{(\alpha + 3\beta_h)^2} - \frac{1}{\beta_e} \right)
$$

(24)

where the superscripted $s$ refers to ‘symmetric.’

Now, assume that one firm anticipates monopolizing the market in the second stage, while the other firm anticipates being shut out of the market. Without loss of generality, assume that $H$ is the monopolizing firm, and that $L$ gets shut out. $L$ clearly will not invest in either type of positioning. $H$ will not invest in horizontal differentiation either but will invest in efficiency improvement up to the point where the marginal benefit of an increase in efficiency, which equals 1, exactly matches the cost. That implies the investment levels $\hat{e}_H^{*H} = (2\beta_e)^{-1}$ and $\hat{e}_L^{*H} = h_H^{*H} = h_L^{*H} = 0$, where the $H$ in the superscript indicates that $H$ monopolizes. The case where $L$ monopolizes is analogous: $\hat{e}_L^{*L} = (2\beta_e)^{-1}$ and $\hat{e}_H^{*L} = h_H^{*L} = h_L^{*L} = 0$. In each case, the monopolizing firm earns a net profit of $(4\beta_e)^{-1}$, and the other firms earns zero net profit.

Since the objective function is not uniformly concave over the range of possible $(\hat{e}_i, \hat{h}_i)$, we used numerical methods to confirm that the symmetric and monopoly solutions are both Nash equilibria (i.e., that neither firm would benefit from unilaterally deviating). In each iteration of our numerical approach, we chose particular values for $\alpha, \sigma, \beta_e, \beta_h$, derived each firm’s positioning for the symmetric and monopoly equilibria at these parameter values, and examined whether one of the firms would have an incentive to unilaterally deviate from the candidate equilibrium to 1 million other possible combinations of horizontal differentiation and efficiency improvement. If neither firm could so profitably deviate, we considered that solution to be a Nash equilibrium. We then
repeated this process \(9^4 = 6,561\) times by allowing each of the four parameters \((\alpha, \sigma, \beta_e, \beta_h)\) to assume every possible combination of nine values.

To rule out the possibility of other asymmetric equilibria, we reduced the four-equation system to two equations in terms of \(h_H\) and \(h_L\). Then, for each of \(9^4 = 6,561\) different combinations of the parameters \((\alpha, \sigma, \beta_e, \beta_h)\), we searched for other solutions among four million different combinations of \(h_H\) and \(h_L\) using a grid pattern. If a particular asymmetric combination of \(h_H\) and \(h_L\) ‘solved’ the system (within a large numerical tolerance\(^{15}\)), our numerical routine investigated whether either firm had a profitable unilateral deviation from that candidate ‘solution’ to, for example, the investment levels of a monopolizing firm. In every such instance, one or both firms had a profitable unilateral deviation.

Because the investment-cost functions are convex and symmetric across firms, any cooperative solution where both firms make at least one type of positive investment must have both firms making symmetric investments. We therefore calculate the sum of the two firms’ profit functions from Equation (3), and then differentiate this sum with respect to both firms’ positioning variables \((e_i, h_i)\). We use the resulting system of four equations in four variables to generate the following symmetric cooperative solution:

\[
\begin{align*}
\bar{h}_H &= \bar{h}_L = \frac{\alpha \sigma}{\beta_h}, \\
\bar{p}_{e}^{\text{ss}} &= \bar{p}_{L}^{\text{ss}} = \gamma + \frac{4 \alpha^2 \sigma^2}{\beta_h}, \\
\bar{\Pi}_{HN}^{\text{ss}} &= \bar{\Pi}_{LN}^{\text{ss}} = \frac{\alpha^2 \sigma^2}{\beta_h}
\end{align*}
\]  

(25)

The bar symbols above the solution variables indicate cooperation, and the \(s\) superscript means ‘symmetric.’ The other way firms might cooperate would be for one firm to spend nothing on positioning and the other firm to position itself to monopolize the market in the second stage. Total industrywide net profits in the symmetric cooperative solution are \(2 \alpha^2 \sigma^2 / \beta_h\) and in the monopoly solution are \((4 \beta_e)^{-1}\). So, the symmetric solution dominates if and only if \(8 \alpha^2 \sigma^2 \beta_e / \beta_h > 1\). Just as in the competitive case, then, the symmetric solution dominates if and only if \(\sigma\) or \(\beta_e\) are large, or \(\beta_h\) is small.

Differentiating Equations (23) to (25) with respect to \(\sigma\) establishes Proposition 1.5:

\[
\begin{align*}
\frac{\partial \bar{h}_H}{\partial \sigma} &= -\frac{\partial \bar{h}_L}{\partial \sigma} = \frac{3 \alpha}{2 (\alpha + 3 \beta_h)} > 0, \\
\frac{\partial \bar{p}_{e}^{\text{ss}}}{\partial \sigma} &= \frac{\partial \bar{p}_{L}^{\text{ss}}}{\partial \sigma} = \frac{12 \alpha^2 \sigma^2}{\alpha + 3 \beta_h} > 0, \quad \text{and} \\
\frac{\partial \bar{\Pi}_{HN}^{\text{ss}}}{\partial \sigma} &= \frac{\partial \bar{\Pi}_{LN}^{\text{ss}}}{\partial \sigma} = \frac{3 \alpha^2 \sigma^2 (4 \alpha + 9 \beta_h)}{2 (\alpha + 3 \beta_h)^2} > 0 \\
\frac{\partial \bar{h}_H}{\partial \sigma} &= -\frac{\partial \bar{h}_L}{\partial \sigma} = \frac{\alpha}{\beta_h} > 0, \\
\frac{\partial \bar{p}_{e}^{\text{ss}}}{\partial \sigma} &= \frac{\partial \bar{p}_{L}^{\text{ss}}}{\partial \sigma} = \frac{8 \alpha^2 \sigma^2}{\beta_h} > 0, \quad \text{and} \\
\frac{\partial \bar{\Pi}_{HN}^{\text{ss}}}{\partial \sigma} &= \frac{\partial \bar{\Pi}_{LN}^{\text{ss}}}{\partial \sigma} = \frac{2 \alpha^2 \sigma^2}{\beta_h} > 0
\end{align*}
\]

(26)\n
Subtracting parts of Equations (23) and (24) from the corresponding parts of Equation (25) yields:

\[
\begin{align*}
\bar{e}_H - \bar{e}_L &= -\frac{1}{6 \beta_e} < 0, \\
\bar{\Pi}_{HN}^{\text{ss}} - \bar{\Pi}_{LN}^{\text{ss}} &= \frac{\alpha (2 \alpha + 3 \beta_h)}{2 \beta_h (\alpha + 3 \beta_h)} > 0, \quad \text{and} \\
\bar{\Pi}_{HN}^{\text{ss}} - \bar{\Pi}_{HN}^{\text{ss}} &= \frac{1}{\beta_e} + \left(\frac{\alpha^2 \sigma^2 (2 \alpha + 3 \beta_h)^2}{4 \beta_h (\alpha + 3 \beta_h)^2}\right) > 0
\end{align*}
\]

(27)\n
(29)

Equations (28) and (29) are the basis for Proposition 1.6. Differentiating Equation (29) and part of Equation (28) with respect to \(\sigma\) yields the following derivatives as the basis for Proposition 1.7:

\[
\begin{align*}
\frac{\partial (\bar{h}_H - \bar{h}_L)}{\partial \sigma} &= \frac{\alpha (2 \alpha + 3 \beta_h)}{2 \beta_h (\alpha + 3 \beta_h)} > 0 \quad \text{and} \\
\frac{\partial (\bar{\Pi}_{HN}^{\text{ss}} - \bar{\Pi}_{HN}^{\text{ss}})}{\partial \sigma} &= \frac{\alpha^2 \sigma^2 (2 \alpha + 3 \beta_h)^2}{2 \beta_h (\alpha + 3 \beta_h)^2} > 0
\end{align*}
\]

(30)

---

\(^{15}\)Because we were looking for solutions using a grid search pattern, solving these equations exactly would only happen by coincidence. We used a large error margin to ensure we did not miss any candidate solutions.

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Model 2: Pricing Subgame

If both firms have positive market shares, there must be a single marginal consumer located at an interior point \( \tilde{\tilde{Y}} \in (\mu - \delta, \mu + \delta) \) who is indifferent between firms \( H \) and \( L \). This occurs where:

\[
v_H \tilde{\tilde{Y}} - p_H = v_L \tilde{\tilde{Y}} - p_L
\]

(31)

Solving for \( \tilde{\tilde{Y}} \) yields:

\[
\tilde{\tilde{Y}} = \frac{p_H - p_L}{v_H - v_L}
\]

(32)

This implies that the equilibrium market shares of each firm are

\[
q_H = \frac{1}{2} + \frac{\mu}{2\delta} \frac{p_H - p_L}{2g} \quad \text{and} \quad q_L = 1 - q_H
\]

(33)

The second-stage gross profit functions are

\[
\Pi_{HG} = q_H (p_H - c_H) = q_H (p_H - \gamma - \omega v_H^2)
\]

and

\[
\Pi_{LG} = q_L (p_L - c_L) = q_L (p_L - \gamma - \omega v_L^2)
\]

(34)

Differentiating each \( \Pi_i \) with respect to its corresponding \( p_i \), setting both derivatives equal to zero, and solving the resulting equation system yields equilibrium prices and margins:

\[
p_L^* = \gamma + \frac{(3\delta - \mu) (v_H - v_L) + \omega (v_H^2 + 2v_L^2)}{3},
\]

\[
p_H^* = \gamma + \frac{(3\delta + \mu) (v_H - v_L) + \omega (2v_H^2 + v_L^2)}{3},
\]

(35)

\[
m_L^* = (p_L^* - c_L^*) = g + \frac{w_L}{3},
\]

\[
m_H^* = (p_H^* - c_H^*) = g + \frac{w_H}{3}
\]

(36)

Equation (36) implies that the rivalry restraint measure \( g \) equals the industry average margin. Substituting these prices into Equations (33) and (34) yields market shares and gross profits of

\[
q_L^* = \frac{m_L^*}{2g} = \frac{1}{2} + \frac{w_L}{6g}, \quad q_H^* = \frac{m_H^*}{2g} = \frac{1}{2} + \frac{w_H}{6g}
\]

(37)

\[
\Pi_L^* = q_L^* m_L^* = \left( \frac{1}{2} + \frac{w_L}{6g} \right) \left( g + \frac{w_L}{3} \right)
\]

\[
\Pi_H^* = q_H^* m_H^* = \left( \frac{1}{2} + \frac{w_H}{6g} \right) \left( g + \frac{w_H}{3} \right)
\]

(38)

This equilibrium is only valid if \( q_L^* > 0 \) and \( q_H^* > 0 \), which is equivalent to Equation (5) or \( |w_i| < 3g \). Assuming this boundary condition is satisfied, the main effects of quality on margin and market share are:

\[
\frac{\partial m_L^*}{\partial v_L} = \frac{\mu - 2\omega v_L}{3} - \delta,
\]

\[
\frac{\partial m_H^*}{\partial v_H} = \frac{\mu - 2\omega v_H}{3} + \delta, \quad \text{and}
\]

\[
\frac{\partial q_H^*}{\partial v_L} = \frac{\omega}{6\delta} > 0
\]

(39)

These derivatives are the basis for Propositions 2.1 and 2.2. In the two margin derivatives, the first term reflects changes in competitive advantage, while the second term reflects changes in rivalry restraint. For simplicity, start with the special case where there is no customer heterogeneity, \( \delta = 0 \), so the rivalry restraint term disappears. In that case, the competitive advantage term shows the trade-off between the benefit of increased customer willingness to pay for the product and its associated higher cost, with margin peaking where this cost and benefit exactly balance each other out, \( \hat{v}_i = \mu/2\omega > 0 \); this is also the level that maximizes competitive advantage \( w_i \). Below this point, quality improvement increases margin, because the higher willingness to pay of customers outweighs the higher variable costs of production; above this point, quality improvement reduces margin, because the higher variable costs outweigh the higher willingness to pay. If there is some customer heterogeneity, \( \delta > 0 \), the level of quality at which margin peaks shifts upward to \( \hat{v}_H = (\mu + 3\delta)/2\omega \) for \( H \) and downward to \( \hat{v}_L = (\mu - 3\delta)/2\omega \) for \( L \). For \( H \), the level of quality at which margin peaks must be positive, so the impact of quality on \( H \)’s margin is always
levels. Quality improvement by reduces competitive advantage at high quality levels and if shows how quality improvement affects profit via competitive advantage, and the second term shows how quality improvement affects profit in both terms are positive, so the net effect rivalry restraint, depends on how quality improvement affects in customer heterogeneity, we differentiate each of the terms in Equations (40) and (41) separately with respect to δ. For L, the derivative of the second term of Equation (40) with respect to δ is $-\left[1/2 + (w_L^2/18g^2)\right]$, which is always negative, while the derivative of the first term is $[-w_L(\mu - 2\omega v_L)]/\delta g$, which has an ambiguous sign but tends toward zero as δ gets large. Combining these two derivatives shows that the effect of customer heterogeneity on the total profitability of quality improvement for L is ambiguous for small values of δ but becomes unambiguously negative at sufficiently large values of δ. For H, the derivative of the second term in Equation (41) with respect to δ is $1/2 + (w_H^2/18g^2)$, which is always positive, while the derivative of the first term is $[-w_H(\mu - 2\omega v_H)]/\delta g$, which has an ambiguous sign but tends toward zero as δ gets large. Combining these two derivatives shows that the effect of customer heterogeneity on the total profitability of quality improvement for H is ambiguous for small values of δ but becomes unambiguously positive at sufficiently large values of δ. These results are the basis for Proposition 2.5.

The externalities of quality improvement in Proposition 2.6 are derived as follows:

$$\frac{\partial \Pi^*_L}{\partial v_L} = \left(\frac{3g + w_L}{9g}\right) \frac{\partial w_L}{\partial v_L} + \left(\frac{(3g + w_L)(3g - w_L)}{18g^2}\right) \frac{\partial g}{\partial v_L}$$ (40)

$$\frac{\partial \Pi^*_H}{\partial v_H} = \left(\frac{3g + w_H}{9g}\right) \frac{\partial w_H}{\partial v_H} + \left(\frac{(3g + w_H)(3g - w_H)}{18g^2}\right) \frac{\partial g}{\partial v_H}$$ (41)

In each of these equations, the first term shows how quality improvement affects profit via competitive advantage, and the second term shows how quality improvement affects profit via rivalry-restraint. Because $|w_L| < 3g$, the fractions in both terms are positive, so the net effect depends on how quality improvement affects rivalry restraint, g, and competitive advantage, $w_i$. The first term is positive if $v_i < \hat{v}_i$ but negative if $v_i > \hat{v}_i$, because quality improvement raises competitive advantage at low quality levels and reduces competitive advantage at high quality levels. Quality improvement by H moves it farther from L and thereby increases rivalry restraint, whereas quality improvement by L moves it closer to H and thereby decreases rivalry restraint. So, the second term is negative in Equation (40) but positive in Equation (41).

Differentiating Equation (39) with respect to δ is the basis for Proposition 2.4:

$$\frac{\partial^2 m^*_L}{\partial \delta \partial v_H} = - \frac{\partial^2 m^*_L}{\partial \delta \partial v_L} = 1 \quad \text{and} \quad \frac{\partial^2 q^*_H}{\partial \delta \partial v_H} = - \frac{\partial^2 q^*_L}{\partial \delta \partial v_L} = \omega \frac{\omega}{\delta^2} > 0 \quad (42)$$

In order to determine how the profitability of quality improvement is affected by customer heterogeneity, we differentiate each of the terms in Equations (40) and (41) separately with respect to δ. For L, the derivative of the second term of Equation (40) with respect to δ is $-\left[1/2 + (w_L^2/18g^2)\right]$, which is always negative, while the derivative of the first term is $[-w_L(\mu - 2\omega v_L)]/\delta g$, which has an ambiguous sign but tends toward zero as δ gets large. Combining these two derivatives shows that the effect of customer heterogeneity on the total profitability of quality improvement for L is ambiguous for small values of δ but becomes unambiguously negative at sufficiently large values of δ. For H, the derivative of the second term in Equation (41) with respect to δ is $1/2 + (w_H^2/18g^2)$, which is always positive, while the derivative of the first term is $[-w_H(\mu - 2\omega v_H)]/\delta g$, which has an ambiguous sign but tends toward zero as δ gets large. Combining these two derivatives shows that the effect of customer heterogeneity on the total profitability of quality improvement for H is ambiguous for small values of δ but becomes unambiguously positive at sufficiently large values of δ. These results are the basis for Proposition 2.5.

The externalities of quality improvement in Proposition 2.6 are derived as follows:

$$\frac{\partial \Pi^*_L}{\partial v_H} = \left(\frac{3g + w_L}{9g}\right) \frac{\partial w_L}{\partial v_H} + \left(\frac{(3g + w_L)(3g - w_L)}{18g^2}\right) \frac{\partial g}{\partial v_H}$$ (43)

$$\frac{\partial \Pi^*_H}{\partial v_L} = \left(\frac{3g + w_H}{9g}\right) \frac{\partial w_H}{\partial v_L} + \left(\frac{(3g + w_H)(3g - w_H)}{18g^2}\right) \frac{\partial g}{\partial v_L}$$ (44)

Setting Equation (43) equal to zero reveals that the local minimum exceeds the local maximum and that any value of $v_H$ that satisfies the boundary condition in Equation (5) must be greater than or equal to the local minimum. Therefore, $\Pi^*_L$ increases monotonically over the possible values of $v_H$. A similar argument shows that $\Pi^*_H$ decreases monotonically over the possible values of $v_L$.

Model 2: Investment subgame

We first consider competitive positioning. Differentiating Equation (38) and setting the resulting
derivatives equal to zero yields a system of two equations in two variables. If \( \delta < 2\mu/3 \), the solution to this system is the Nash equilibrium. If \( \delta \geq 2\mu/3 \), the solution to this system would imply a negative quality level for \( L \), which has been ruled out by assumption. In that case, the Nash equilibrium has \( L \) set \( v_L = 0 \), and \( H \) choose its own quality level as the best response to \( v_L = 0 \). In either case, the second-order conditions are satisfied. Combining both cases, we have:

\[
\begin{align*}
\bar{v}_L &= \begin{cases} 
(2\mu - 3\delta)/4\omega & \text{if } \delta < 2\mu/3 \\
0 & \text{if } \delta \geq 2\mu/3 
\end{cases} \\
\bar{v}_H &= \begin{cases} 
(2\mu + 3\delta)/4\omega & \text{if } \delta < 2\mu/3 \\
(\mu + 3\delta)/3\omega & \text{if } \delta \geq 2\mu/3 
\end{cases}
\end{align*}
\]

Substituting this into Equations (37) and (38) yields equilibrium market shares, margins, and profits:

\[
\begin{align*}
\tilde{q}_L &= \begin{cases} 
1/2 & \text{if } \delta < 2\mu/3 \\
(6\delta - \mu)/9\delta & \text{if } \delta \geq 2\mu/3 
\end{cases} \\
\tilde{q}_H &= \begin{cases} 
1/2 & \text{if } \delta < 2\mu/3 \\
(3\delta + \mu)/9\delta & \text{if } \delta \geq 2\mu/3 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\tilde{m}_L &= \begin{cases} 
3\delta^2/2\omega & \text{if } \delta < 2\mu/3 \\
2(6\delta - \mu)(3\delta + \mu)/27\omega & \text{if } \delta \geq 2\mu/3 
\end{cases} \\
\tilde{m}_H &= \begin{cases} 
3\delta^2/2\omega & \text{if } \delta < 2\mu/3 \\
2(3\delta + \mu)^2/27\omega & \text{if } \delta \geq 2\mu/3 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\Pi_L^* &= \begin{cases} 
3\delta^2/4\omega & \text{if } \delta < 2\mu/3 \\
2(3\delta + \mu)(6\delta - \mu)^2/243\omega & \text{if } \delta \geq 2\mu/3 
\end{cases} \\
\Pi_H^* &= \begin{cases} 
3\delta^2/4\omega & \text{if } \delta < 2\mu/3 \\
2(3\delta + \mu)^3/243\omega & \text{if } \delta \geq 2\mu/3 
\end{cases}
\end{align*}
\]

Proposition 2.7 is based on the following effects of \( \delta \) on the equilibrium outcomes shown above:

\[
\begin{align*}
\frac{\partial v^*}{\partial \delta} &= \begin{cases} 
-3/4\omega & \text{if } \delta < 2\mu/3 \\
0 & \text{if } \delta \geq 2\mu/3 
\end{cases} \\
\frac{\partial \Pi_H^*}{\partial \delta} &= \begin{cases} 
3/4\omega & \text{if } \delta < 2\mu/3 \\
1/\omega & \text{if } \delta \geq 2\mu/3 
\end{cases}
\end{align*}
\]

If the firms cooperate in competitive positioning, they will never choose quality levels that result in both firms having positive market shares. To prove this, we add together the two firms’ second-stage equilibrium profit functions from Equation (38) to get the industry-wide total profit if both firms have positive market shares. This equation has no maximum where \( 0 \leq v_L < \infty, 0 \leq v_H < \infty \). If \( H \) chooses a quality level of \( +\infty \) and \( L \) chooses any nonnegative quality level, then Equation (5) would be violated, which would mean that both firms would not have strictly positive market shares. Suppose, instead, that \( H \) monopolizes the market in the second stage. The lowest price that \( L \) can charge is its marginal cost of \( \gamma + \omega v^2_L \), so \( H \) can charge a price of no more than \( \gamma + \omega v^2_L + (\mu - \delta)(v_H - v_L) \) in order to make the least quality-sensitive customer (i.e., the customer at \( Y = \mu - \delta \)) indifferent between the two firms. \( L \) will then have market share and profit of zero, while \( H \) has a market share of one and profit of:

\[
\Pi_H^* = v_H (\mu - \delta) - \omega v^2_H - [v_L (\mu - \delta) - \omega v^2_L]
\]

This equation is clearly bounded for any nonnegative value of \( v_L \).
Now, suppose $L$ monopolizes in the second stage. The lowest price that $H$ can charge is its marginal cost of $\gamma + \omega v_H^2$, so $L$ can charge a price of no more than $\gamma + \omega v_H^2 - (\mu + \delta) (v_H - v_L)$ in order to make the most quality-sensitive customer (i.e., the customer at $Y = \mu + \delta$) indifferent between the two firms. $H$ will then have a market share and profit of zero, while $L$ has a market share of one and profit of:

$$\Pi_L^L = [v_L (\mu + \delta) - \omega v_L^2] - [v_H (\mu + \delta) - \omega v_H^2]$$

(54)

This profit can be made arbitrarily large by setting $v_L = (\mu + \delta)/2\omega$ to maximize the first bracketed term for any finite level of $v_H$, and by letting $v_H$ grow arbitrarily large. This result is the basis for Proposition 2.8.